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Crossing the Motorway in a Wheelchair

- Did any new jobs come in last night?
- Let me check. Only one, as far as I can see.
- What is it?
- The Road Department emailed a request. Here it is.

> *To: The Math Team*
> *From: The Road Department*
>
> *Hi team!*
>
> *We are planning a walkway over a couple of motorways. However, instead of doing as in the past, building a staircase from the pavement to the walkway, we would like to build a ramp. That way people with strollers or in wheel chairs can use the walkways too.*
>
> *The ramp will be more expensive to build than a staircase. That is why we are contacting you. Please find the shortest ramp we can use.*
>
> *If you need more information please don't hesitate to ask.*
>
> *Two images are attached.*
>
> *Yours sincerely,*
> *Donald Veivals*

- They have done it again!
- What do you mean?
- Why can't the Road Department be more precise?! To start with, they should have told us how high up the walkways will be.
- We can always call them.
- OK. Please call.

(a bit later)

- What did they say?
- That the height would be between 6 and 8m. At this stage they couldn't be more precise.
- I see.
- Well, lets get to work.
- Let's have a look at the pictures they sent.

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- Interesting problem! The question is, how long will the ramp be?
- Yes, but I am hungry. Let's go to the canteen to grab something to eat. Maybe the grey cells will work while we are eating.

(several bites later)

- That was delicious! What was it called?
- Smoked salmon with pasta.
- I never knew you could mix fish and pasta!
- While we ate I couldn't stop thinking of the problem.
- You have to learn to relax! You should take up meditation. Or yoga.
- How steep can the ramps be?
- OK. Back to work. Well, if they are too steep, a wheel chair can't ascend, and if they are almost flat, they will be extremely long.
- And expensive!
- And expensive.
- What is a good compromise?
- We could call the Road Department again. Maybe they have also thought about the steepness.

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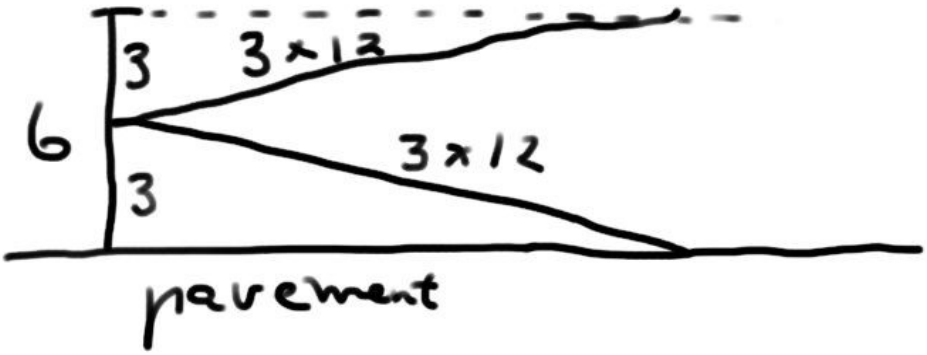
- We could, but they will maybe answer as before. "We do not know at this stage, but we ..." (mimicking the voice of Donald Veivals).
- Let's search the Internet! Let's search for "steepnes of ramps."
- OK.

> *Wheelchair ramps for public access are required to be no steeper than a 1:12 ratio. This means for every one foot of height, the ramp must be 12 feet long.*
>
> http://www.livestrong.com/article/22241-steep-should-wheelchair-ramp/

- Where on earth did you find that?
- It was the third search result. The page is called "How Steep Should a Wheelchair Ramp Be?"
- Exactly what we are looking for!
- Let me draw.



- If the height of the walkway is 6m. The first leg of the ramp would be 3 x 12m = 36m. So the total length of the ramp will be twice that, i.e. 72m.
- And what if the height is 8m?
- 12 x 8 = 96 and twice of that is 192m. What a difference! I am surprised!
- So am I.
- Oh, I am sorry! I did it wrong. Let me try again.
- OK. No problem.
- 12 x 4 = 48 so the total is 96m. That makes a bit more sense. The difference is 24m or 2 x 12.
- That makes sense since 8m is 2m more than 6m.
- What if the height is neither 6 nor 8?
- We could call it h.
- h for height?
- Right. We will let h be a variable and see what happens.
- That is not too difficult. The double of 12 * (h/2) is 12 * h.
- I feel a bit stupid.
- Join the club!
- 12 * 6 = 72 and 12 * 8 = 96. Why didn't I see that earlier?
- It helps when you play with the problem. You see it clearer and clearer.
- I like to see it clearly from the start!
- And I like chocolate cake in bed every morning!
- But that is impossible, right?
- My parents believe so.

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- Back to the problem. What if the steepness is something else than 1:12?
- You mean 1:s? Where s stands for steepness
- Exactly! Then what is the length of the ramp?
- It is easy. Instead of 12 * h it is s * h.
- You mean sh?
- If you like. Although it sounds like you are wanting me to be quiet.
- What do you mean?
- sssshh!
- Very funny. Not!

(a few hours later)

- I read something interesting on the Internet.
- Please share.

> *I have designed ramps for over 3,000 people and the majority of them are surprised to learn that the ramp has to be longer than they expected. Some of them believed they really could put a piece of plywood over their existing steps and have a usable ramp. In my experience, people do not have the information needed to know how long their ramp should be. They understandably want to save money and space, and some have installed ramps that were too steep. They learned the hardway that too steep a ramp is no help at all.* - Bob Zimmerman, *Minnesota Ramp Project*.
>
> http://www.newdisability.com/wheelchairramp.htm

- By the way, have you written a reply to the Road department?
- Would you like to see it?
- Yes, please.

> *To: The Road Department*
> *From: The Math Team*
>
> *Thank you for an interesting problem!*
>
> *The length of the ramp will depend on two things, the height of the walkway and the steepness of the ramp.*
>
> *If the height is h and the steepness is 1:s, where s is the length of the ramp for every meter it is elevated, the total length of the ramp will be hs.*
> *It is suggested on the Internet that s should be 12 and in that case the ramp will be 12h long.*
>
> *I hope this answers your question to your satisfaction.*
>
> *Regards,*
> *Anna*
> *(for the Math Team)*

(the next day)

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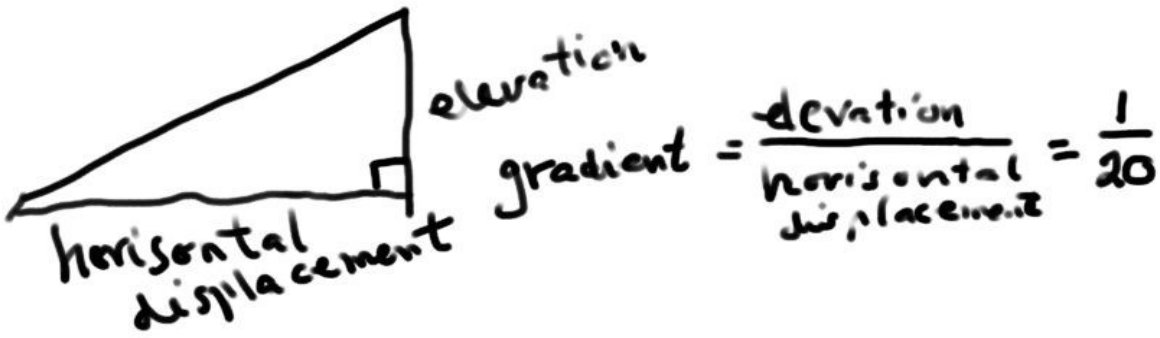
- Did you see the email from the Road Department.
- Sure did. Those guys are unbelievable!

> *To: The Math Team*
> *From: The Road Department*
>
> *Hi team!*
>
> *I am afraid your way to measure steepness is not the one we use. I guess I should have told you this earlier. Sorry!*
>
> *We use the gradient, which is the elevation divided by the horisontal displacement. Last night the City Council told us that a gradient of 1:20 is the maximum allowable for independent wheelchair users.*
>
> *Kindly estimate the length of the ramp again.*
>
> *As this is becoming a matter of urgency I look forward to your reply by the end of today, if possible.*
>
> *Best wishes,*
> *Donald*

- Did you know they measured steepness that way? I had no idea.
- Neither had I.
- Let me draw.



- I think Pythagoras will give us the length of the ramp.
- You mean the one that says c^2 = a^2 + b^2 in a right-angle triangle

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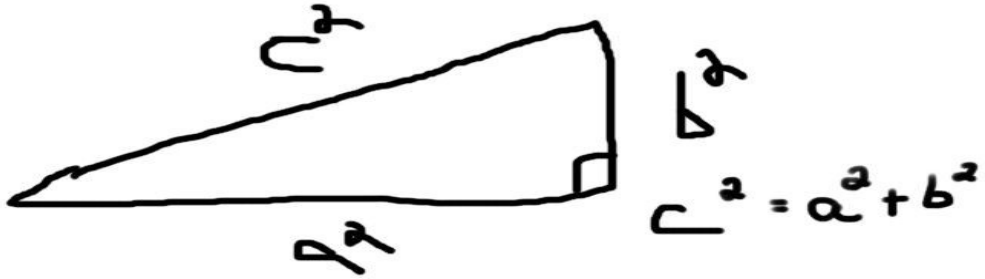


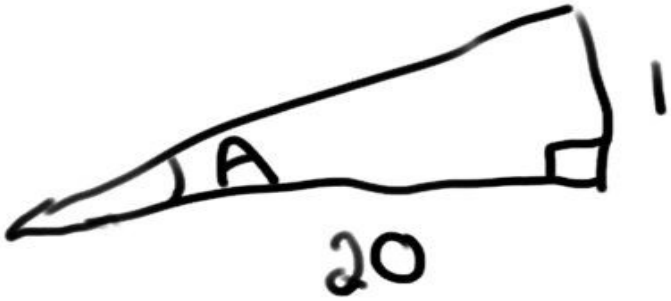
- Right. c^2 = 1^2 + 20^2 = 401. So the length needed to elevate 1m is sqrt(401) = 20.025m.
- Making the total length h x 20.025. Much longer than what we first suggested!
- Send Donald an email straight away. This should make him happy. :)

(a few hour later)

- Donald has sent us a new email.
- A thank you letter I assume?
- Kind of.

> *To: The Math Team*
> *From: The Road Department*
>
> *Hi team!*
>
> *Thank you for the new formula. It will be very useful to us!*
>
> *I have another related favour to ask you. To measure steepness we use a tool called the inclinometer. This device measures steepness not in gradient, but in degrees. We know the gradient should be 1:20, but how much is that in degrees? Maybe I am asking too much, but it would be nice to know how to find the angle for any gradient.*
>
> *I have confidence in you guys. Don't let us down!*
>
> *Best wishes,*
> *Donald Veivals*

- Have you noticed that he signed with his full name again. He does that when he has a serious problem!



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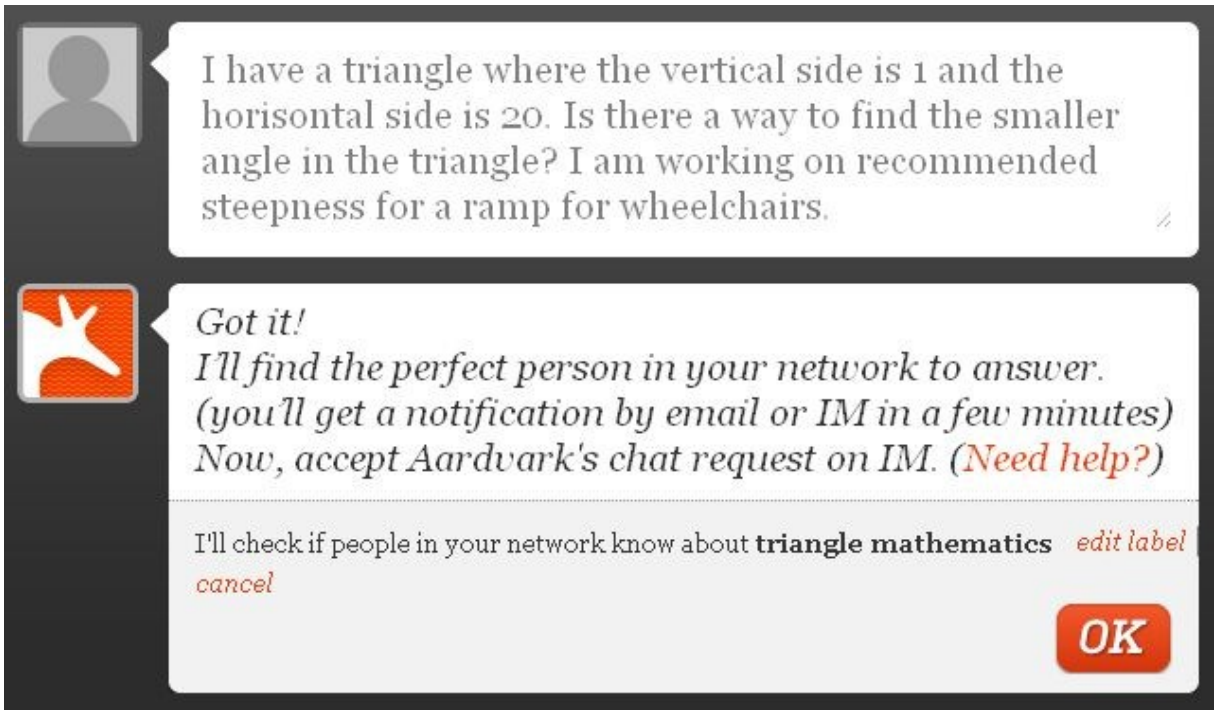
- How can we find the angle A?
- We could draw a big triangle where the ratio is 1:20 and then measure.
- OK let's try that.

(a few minutes later)

- What did you get?
- 2 degrees.
- And you?
- 4.5 degrees.
- I have a feeling Donald will like a much more precise answer!
- Aardvark!
- You are saying?
- Let's ask Aardvark.
- Does he live close to here?
- It is not a he! It is a help system for curious minds. You can ask anything and volunteers will respond in minutes.
- You are pulling my leg!?
- Far from it. Let me show you.



- Amazing! Do you think anyone will answer?
- Let's wait and see.

(10 minutes later)

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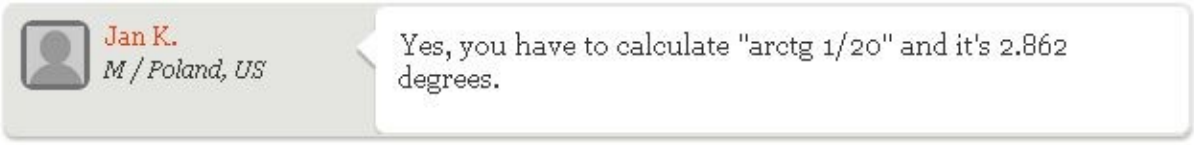
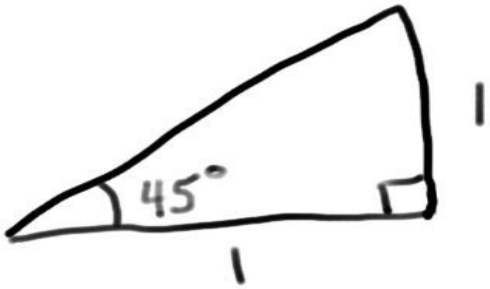
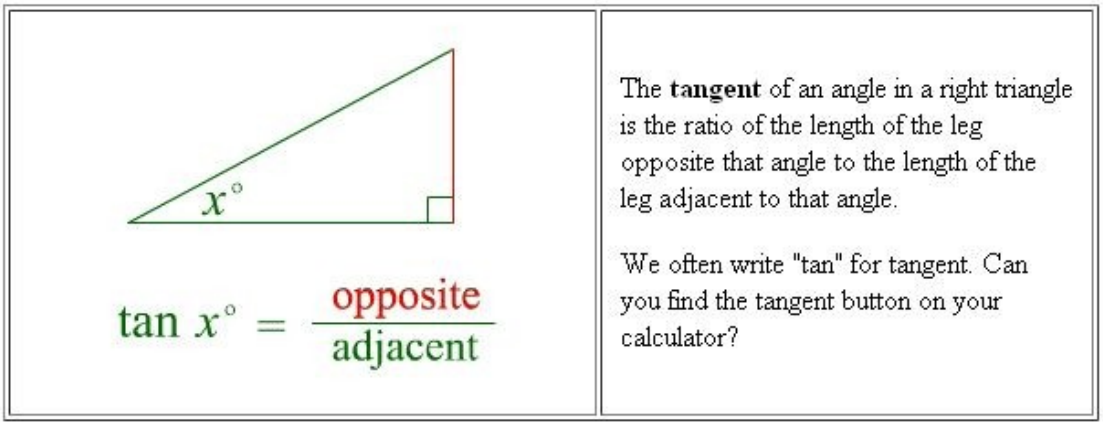


- I don't know which rules Jay talk about. What is this arctg Jan mentions?
- Let me look at the manual for my calculator.
- Do I have time to write a novel meanwhile?
- No! Here it is. "For the inverse of tan (argtan) use tan^-1."
- Do you know how?
- I have seen the button, but I don't know how to use it. Let's try it. I press Shift tan^(-1) (1 / 20) =.
- I can't believe it! It got 2.862405226. A bit more precise than 2 and 4.5!
- Let's try it for other triangles. Like 1:10, 1:5, 1:2, 1:1.
- For 1:1 the triangle is isosceles.



- The angle should be 45.
- Indeed tan^(-1)(1/1) = 45!
- I feel like Superman! Thinking a bit, searching the Internet, asking Arvark, and talking to you has made me invincible!
- Good for you. I don't feel quite as powerful. What is tan anyway?
- Search for it on the Internet.
- Let me try: "tan button on calculator."



http://www.ferrismath.com/int1/Sine,%20Cosine%20and%20Tangent.htm

- Try tan of 2.862405226. It should give 1/20?
- It gave 0.05.

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- Press the a b/c button.
- It gives 1/20. It works!
- That is amazing. If we know two sides we can find the angle! I wonder how the tan and tan^(-1) button works.
- What do you mean. You saw how it works, didn't you. It finds the ratio or the angle.
- But how? How does it find them?

(the next day)

- Last night my father told me that when he was a boy, there were no calculators, no Internet, no computers...
- Were there dinosaurs?!
- I didn't ask him, but I doubt it.
- So your father was born before the dinosaurs. Impressive!
- He had a tan table. A table of values for tan. He found it in the attic. Here is some of it.

Rad	Deg	Sin	Cos	Tan	Csc	Sec	Cot		
.0000	00	.0000	1.0000	.0000	-----	1.0000	-----	90	1.5707
.0175	01	.0175	.9998	.0175	57.2987	1.0002	57.2900	89	1.5533
.0349	02	.0349	.9994	.0349	28.6537	1.0006	28.6363	88	1.5359
.0524	03	.0523	.9986	.0524	19.1073	1.0014	19.0811	87	1.5184
.0698	04	.0698	.9976	.0699	14.3356	1.0024	14.3007	86	1.5010
.0873	05	.0872	.9962	.0875	11.4737	1.0038	11.4301	85	1.4835
.1047	06	.1045	.9945	.1051	9.5668	1.0055	9.5144	84	1.4661
.1222	07	.1219	.9925	.1228	8.2055	1.0075	8.1443	83	1.4486

http://math2.org/math/trig/tables.htm

- Very confusing! Your father must have been a genius!
- He still thinks he is...
- I know what you mean!
- I don't know what Rad, Sin, and other columns stand for, but if you look in the Tan column you find .0524 which is close to 0.05 or 1/20.
- I got it! In the Deg column it says 03. I guess that is the angle we were looking for. It is not very accurate though. It is a long way from the accuracy of 2.862405226.
- How on earth does the calculator finds so many digits. Do you think it is storing a gigantic table?
- Could be.
- What else can it do?
- It can maybe calculate tan somehow.
- But how?
- How many emails have we sent to the Guru this month?
- Four. We have one left we can send him before we reach the limit of five per month.
- Do you think it is worth asking him? You never know if we needs his help later this month. There are still eleven days left.

- I don't know if it is worth it. Besides, the emails from the Guru are sometimes very difficult to decipher. However, he answers almost immediately. Let's flip a coin to decide.
- OK. Head for calling him. Let me flip.
- Head it is. Let's ask him.

(a few hours later)

- The Guru's answer looks like a riddle.

```
def arctan( x ):
if abs(x) >= 1:
  return 'I cannot find the angle. Sorry!'
else:
   radians = 0
    lastdegrees = 0
    n = 0
    while True:
      radians = radians + (pow(-1, n) * pow(x, 2*n+1)) / (2*n + 1)
     degrees = 180 * radians / 3.1415926535897932
      if abs(degrees - lastdegrees) < 1e-9:
        break
      n = n + 1
      print n, degrees
      lastdegrees = degrees
return 'The angle is ' + str(degrees) + ' degrees.'
```

- Let's try it!
- Copy the code into the Python editor and type arctan(0.05).
- Look at the result!

```
>>> arctan(0.05)
1 2.86478897565
2 2.86240165151
3 2.86240523249
4 2.8624052261
'The angle is 2.86240522611 degrees.'
>>>
```

- Impressive!
- But I don't have a clue what the program does.
- Same here. But I think it needed four attempts to get to the answer.
- Impressive!

--- fin ---

Exercises: (left to the reader to create/discover/invent and solve/ponder/investigate)